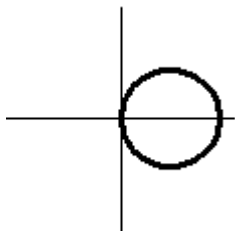


HOW TO GRAPH POLAR FUNCTIONS OF THE FORM $r = a \pm b \cos \theta$ OR $r = a \pm b \sin \theta$

- Determine the shape of the graph by finding $\left| \frac{a}{b} \right|$.

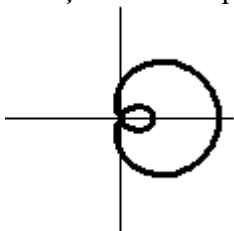
$$\left| \frac{a}{b} \right| = 0$$

circle



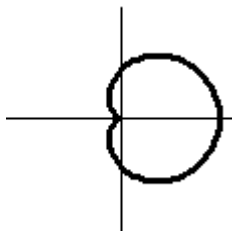
$$\left| \frac{a}{b} \right| < 1$$

limaçon with loop



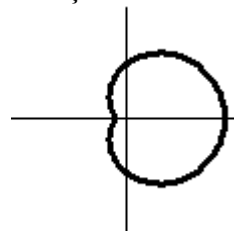
$$\left| \frac{a}{b} \right| = 1$$

cardioid



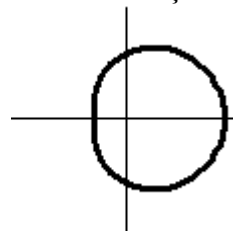
$$1 < \left| \frac{a}{b} \right| < 2$$

limaçon with dimple



$$\left| \frac{a}{b} \right| \geq 2$$

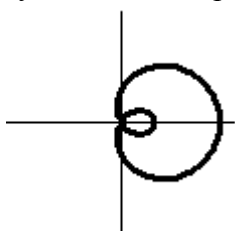
convex limaçon



- Determine the axis of symmetry by the trigonometric function used.

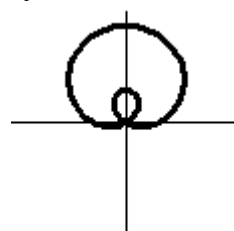
$$r = a \pm b \cos \theta$$

symmetric over polar axis (x - axis)



$$r = a \pm b \sin \theta$$

symmetric over $\theta = \frac{\pi}{2}$ (y - axis)

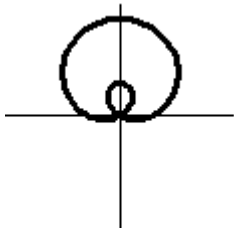
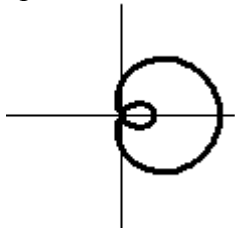


- Determine the “direction” of the graph by the sign of b .

$$b > 0$$

“bigger” end on the right/top

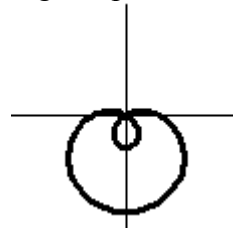
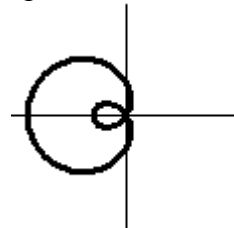
“puckered” end on the left/bottom



$$b < 0$$

“bigger” end on the left/bottom

“puckered” end on the right/top



- Determine the x - and y - intercepts by finding the points corresponding to $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

$$\theta = 0, \pi$$

correspond to positive and negative x - intercepts (assuming $r > 0$)

$$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

correspond to positive and negative y - intercepts (assuming $r > 0$)

If $r < 0$, the intercept is on the “other side” (negative vs positive, and vice versa) of the corresponding axis

Example: Graph $r = 2 - 3 \sin \theta$

1.

$\left| \frac{2}{-3} \right| < 1$

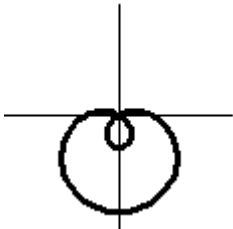
limaçon with loop
2.

equation uses $\sin \theta$

symmetric over $\theta = \frac{\pi}{2}$ (y – axis)
3.

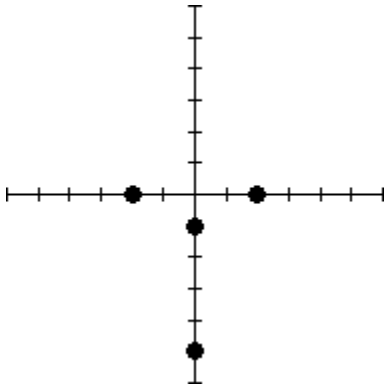
$-3 < 0$

“bigger” end on the bottom, “puckered” end on the top
4.



θ	$r = 2 - 3 \sin \theta$
0	2 (positive x – intercept)
$\frac{\pi}{2}$	-1 (negative y – intercept)
π	2 (negative x – intercept)
$\frac{3\pi}{2}$	5 (negative y – intercept)

Intercepts only



Complete graph

